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Dynamic response of Timoshenko beam under moving mass

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Dynamic amplification.

Abstract In this article, the dynamic responses of a Timoshenko beam subjected to a moving mass, and a moving sprung mass are analyzed. By making recourse to Hamilton's principle, governing differential equations for beam vibration are derived. By using the modal superposition method, the partial differential equations of the system are transformed into a set of Ordinary Differential Equations (ODEs). The resulted set of ODEs is represented in state-space form, and solved by means of a numerical technique. The accuracy of the results has been ascertained through comparing the results of our approach with those available from previous studies; moreover, a reasonable agreement has been obtained. The quantities of the dynamic response of the beam for the case of moving sprung mass are confronted with moving mass and also moving load cases. Through extensive numerical campaign, it is concluded that with respect to the applied values of suspension system properties, the deflections of the beam subjected to moving load case are an upper bound for moving sprung mass results.

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1. Introduction

The interaction of moving bodies and structural systems is a major research field in the realm of the structural engineering. In recent years, by developing high speed train lines, a renewed interest toward developing numerical techniques to solve differential equations for beam vibration under moving force can be observed; nevertheless, closed form and numerical solutions for moving body problems are sought by the researchers [1–3].

The early studies on this issue were mostly focused on providing closed-form solutions for the problem. Therefore, Timoshenko [4] made classic contributions where he solved the problem of a simply supported beam subjected to a moving force by using the power series expansion. In addition, Jeefcot [5] investigated the problem with less idealization, by considering both the moving mass and beam mass inertia which led to solving the equation by applying an iterative method. Inglis [6] performed a thorough investigation of the dynamic response of the railway bridges and assessed the effect of important factors such as the suspension system of the train. Fryba [7] performed a thorough study of vibration for a simply supported beam subjected to a variety of traveling loads.

Evolution of microcomputers caused an unavoidable interest toward discrete analysis techniques [8–11]. These discrete techniques, especially finite element methods, were extensively employed to solve governing differential equations of systems. Olsson [12] investigated the interaction of bridge and vehicle by deriving a general bridge-vehicle element. The element was regarded as a finite element with time-dependent and asymmetric element matrices. Mofid et al. [13,14] developed discrete element technique to solve the problem of dynamic response of Euler–Bernoulli beams subjected to moving mass. It is a fast and simple method to solve this problem. Yavari et al. [15] further worked on the discrete element method, by considering shear deformation and rotary inertia of the beam. Green and D. Cebon [16] studied the bridge-vehicle interaction by considering a vehicle model as a lumped mass supported by spring and damper. By using iterative method, they analyzed a limited range of sprung mass properties. Akin

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Nomenclature

I:	beam moment of inertia
A:	beam cross sectional area
ρ:	beam mass per unit volume
g:	acceleration of gravity
m:	lumped moving mass
$w(x,t)$:	transverse dynamic deflection of beam
$u(x,t)$:	horizontal component of displacement field of beam
$X(t)$:	equation of motion of mass over the beam
$h(x,t)$:	vibration of moving sprung mass
$\theta(x,t)$:	angle of rotation of beam cross section with respect to vertical axes
E:	modulus of elasticity
G:	shear modulus
k:	stiffness of suspension system
c:	damping of suspension system
κ:	shear correction factor
p:	angular natural frequency of vibration of the beam.

and M. Mofid [17] presented an analytical–numerical method based on the modal superposition technique. Their approach can easily be applied to determine the dynamic response of beams with various boundary conditions subjected to a moving mass. Lee [18] used the approach developed by Akin to solve the problem of moving mass on Timoshenko beam; furthermore, he investigated the possible separation of the beam and the mass.

In this article, by applying Hamilton's principle, the partial differential equations of a continuous beam subjected to a moving mass and moving sprung mass are derived. By using the modal superposition method, the resulted partial differential equations (PDEs) are transformed into a set of ODE. It is assumed that the speed of moving mass can vary in time. The resulted ODEs are represented in state-space form, and solved by means of a numerical technique. Due to the weakness of Bernoulli theory in analyzing vibration of higher modes [19], especially in long span beams resting on elastic foundation, Timoshenko beam theory is adopted. In this study, the accuracy of the numerical technique adopted has already been assessed in the existing literature, where capabilities of the procedure adopted in this paper in predicting responses of damaged and undamaged beams to moving mass [20] and moving sprung mass [21] are experimentally evaluated.

Mathematical formulation of moving mass and moving sprung mass problems are both more complicated compared to the moving load problem. In the existing literature, many authors have investigated the circumstances in which the moving load formulation can be reasonably used instead of the moving mass formulation, e.g. see [22]. Pesterev and co-authors have investigated asymptotical equivalence of moving mass, moving sprung mass and moving load problems [23]. Furthermore, they concluded that for very large values of stiffness of the spring-dashpot system, the moving sprung mass system is equal to the moving mass system while for small values of the stiffness of the spring-dashpot system, the problem reduces to moving load problem [23]. Despite the fact that in many real-life problems one has to deal with the moving sprung mass formulation, to the best of our knowledge, no parametric study has been carried out in order to determine the range of parameters in which the moving load formulation can yield a reasonable approximation of the moving sprung mass system. To fill this gap, in this paper, the dynamic response of the beam for the case of moving sprung mass is compared with moving mass and moving load cases. Hence, four non-dimensional parameters are varied and their effect on the dynamic response calculated via numerical analysis of the structure is scrutinized. It is observed that with respect to the applied range of suspension system properties, moving load results are an upper bound for moving sprung mass results.

2. Problem formulation

A simply supported Timoshenko beam, as shown in Figure 1 is subjected to a moving mass (or a moving sprung mass) external loading. The velocity of the travelling mass can be variable. A coordinate system is assumed to be fixed in an inertial frame, where the horizontal axis is parallel to undeformed longitudinal axis of the beam, whereas the vertical one points upward. The variables are presented as follows: $X(t)$ defines the projection of the place of moving mass on the horizontal axis; $h(X,t)$ denotes the vertical displacements of moving sprung mass, as seen in Figure 2; $u(x,t)$ represents the horizontal component of displacement field of the body of the beam; $w(x,t)$ is the vertical component of the displacement field of the body of the beam and $\theta(x,t)$ stands for the rotation angle of the beam cross section with respect to the vertical axes. Using the first order beam shear deformation theory, the vertical component represents deflection while the horizontal component is expressed as:

$$u(x,t) = -z\theta(x,t). \tag{1}$$

The partial differential equations of the beam acted by moving mass can be derived by applying Hamilton's principle. For the sake of the brevity, the damping ratio of the beam is not included in the formulation. Kinetic and potential energies along with the work done by the non-conservative forces applied to the system can be expressed as below:

$$\begin{aligned}\Theta(t) &= \frac{1}{2}\int_0^l\int_A\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2\right]\rho\,dA\,dx \\ &\quad+\frac{1}{2}m\,\dot{w}^2\Big|_{x=X}+\frac{1}{2}m\dot{X}^2,\end{aligned} \tag{2}$$

$$\Xi(t) = \frac{1}{2}\int_0^l\int(\sigma_{xx}\varepsilon_{xx}+2\tau_{xz}\varepsilon_{xz})\,dA\,dx - m\,g\,w|_{x=X}, \tag{3}$$

$$\Upsilon(t) = fX. \tag{4}$$

Considering the above notation, the total energy or the Hamiltonian of the system reads [24] as:

$$\pi = \int_{t_1}^{t_2}(\Theta-\Xi+\Upsilon)dt. \tag{5}$$

By performing the first variation with respect to X, θ and w the set of partial differential equations of motion can be derived as:

$$\rho Aw\,w_{tt}+\kappa GA\,(\theta_x-w_{xx}) = (mg-m\ddot{w})\,\delta(x-X), \tag{6}$$

$$\rho I\,\theta_{tt}-EI\,\theta_{xx}+\kappa GA\,(\theta-w_x) = 0, \tag{7}$$

$$m\,X_{tt} = f, \tag{8}$$

where the full derivative of acceleration of moving mass in expanded form is expressed as:

$$\ddot{w}(X,t) = \frac{\partial^2 w}{\partial t^2}+2\dot{X}\frac{\partial^2 w}{\partial x\partial t}+\dot{X}^2\frac{\partial^2 w}{\partial x^2}+\ddot{X}\frac{\partial w}{\partial x}. \tag{9}$$

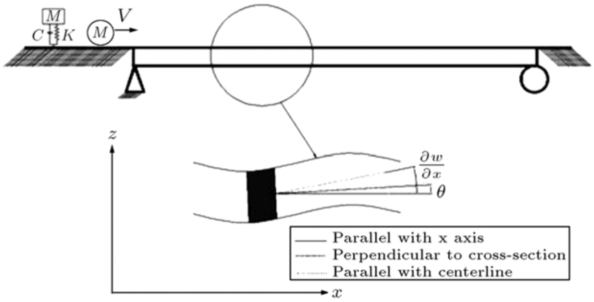

Figure 1: Simply supported Timoshenko beam, carrying moving mass.

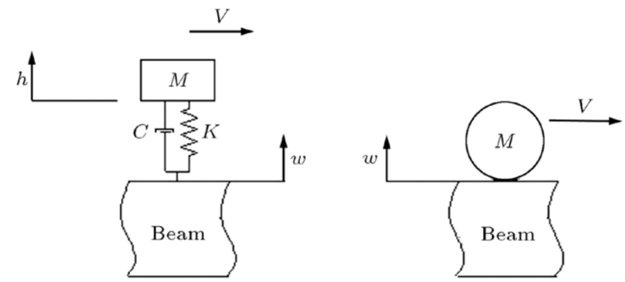

Figure 2: Schematic views of moving sprung mass (left) and moving mass (right).

In the case of the moving sprung mass, kinetic energy, potential energy and the work done by the external non-conservative forces take the form as below:

$$\Theta(t) = \frac{1}{2}\int_0^l \int_A \left[\left(\frac{\partial w}{\partial t}\right)^2 + \left(\frac{\partial x}{\partial t}\right)^2\right] \rho dAdx + \frac{1}{2}m\dot{X}^2 + \frac{1}{2}m\dot{h}^2\Big|_{x=X}, \tag{10}$$

$$\Xi(t) = \frac{1}{2}\int_0^l \int_A (\sigma_{xx}\varepsilon_x + 2\,\tau_{xz}\varepsilon_{xz})\, dAdx - mgh|_{x=X} + \frac{1}{2}k(w|_{x=X} - h|_{x=X})^2, \tag{11}$$

$$\Upsilon(t) = -c\,(w|_{x=X} - h|_{x=X})\,(\dot{w}|_{x=X} - \dot{h}|_{x=X}) + f\,X. \tag{12}$$

For this case, the set of governing partial differential equations of the system is obtained as:

$$\rho A\, w_{tt} - \kappa GA(w_{xx} - \theta_x) = (k(h-w) + c(\dot{h} - \dot{w}))\delta(x - X), \tag{13}$$

$$EI\,\theta_{xx} + \kappa GA\,(w_x - \theta) - \rho I\,\theta_{tt} = 0, \tag{14}$$

$$m\,\ddot{h}|_{x=X} + c\,\dot{h}|_{x=X} + k\,h|_{x=X} = mg + k\,w|_{x=X} + c\,\dot{w}|_{x=X}, \tag{15}$$

$$m\ddot{X} - f = 0, \tag{16}$$

while the expanded form of the full derivatives is written as:

$$\ddot{h}(x,t) = \frac{\partial^2 h}{\partial t^2} + 2\dot{X}\frac{\partial^2 h}{\partial x \partial t} + \dot{X}^2\frac{\partial^2 h}{\partial x^2} + \ddot{X}\frac{\partial h}{\partial x}, \tag{17}$$

$$\dot{h}(X,t) = \frac{\partial h}{\partial t} + \dot{X}\frac{\partial h}{\partial x}, \tag{18}$$

$$\dot{w}(X,t) = \frac{\partial w}{\partial t} + \dot{X}\frac{\partial w}{\partial x}. \tag{19}$$

Dealing with Eq. (17), the convective terms of the derivative are frequently neglected [20], i.e. $\ddot{h}(X,t) \approx \frac{\partial^2 h}{\partial f^2}$; however, in this paper, to analyze the inertial effects of the moving mass all term in Eq. (17) are included in the analysis.

Using the modal superposition method, θ, h and w can be represented as:

$$w = (x,t) = \sum_{i=1}^{\infty} \Phi_i(x)T_i(t), \tag{20}$$

$$\theta(x,t) = \sum_{i=1}^{\infty} \Psi_i(x)Q_i(t), \tag{21}$$

$$h(x,t) - \sum_{i=1}^{\infty} \Phi_i(x)H_i(t), \tag{22}$$

where $\Phi_i(x)$ and $\Psi_i(x)$ are designated as deflection and rotational shape functions of the beam. At first, the case of moving mass without suspension system is considered. By substituting Eqs. (20) and (21) in Eq. (6), and then by multiplying Eq. (6) by $\Phi_j(x)$ and Eq. (7) by $\Psi_j(x)$ and integrating over the interval of $(0, l)$ with respect to the x, the set of coupled partial differential equations are transformed into a set of ordinary differential equations:

$$\boldsymbol{M}\ddot{\boldsymbol{T}} + \boldsymbol{C}\dot{\boldsymbol{T}} + \boldsymbol{K}\boldsymbol{T} + \boldsymbol{\Delta}\,\boldsymbol{Q} = \boldsymbol{F}, \tag{23}$$

$$\overline{\boldsymbol{M}}\ddot{\boldsymbol{Q}} + \overline{\boldsymbol{\Delta}}\boldsymbol{Q} + \overline{\boldsymbol{K}}\boldsymbol{T} = \overline{\boldsymbol{F}}, \tag{24}$$

in which the matrix elements are expressed as the following form:

$$\begin{aligned}
&M_{ij} = \rho A\zeta_{ij} + m\Phi_i(X)\Phi_j(X),\\
&C_{ij} = 2\dot{X}\, m\Phi_i'(X)\Phi_j(X),\\
&K_{ij} = \kappa GA\varrho_{ij} + m\dot{X}^2\Phi_j''(X)\Phi_j(X) + m\ddot{X}\Phi_j'(X)\Phi_j(X),\\
&\Delta_{ij} = \kappa GA\upsilon_{ij},\\
&F_i = mg\Phi_i(X), \qquad \overline{M}_{ij} = \rho I\varsigma_{ij},\\
&\overline{\Delta}_{ij} = \kappa GA\varsigma_{ij} - EI\eta_{ij}, \qquad \overline{K}_{ij} = -\kappa GA\varepsilon_{ij},\\
&\overline{F}_i = 0,
\end{aligned} \tag{25}$$

where to obtain a compact form, these coefficients are defined as:

$$\begin{aligned}
&\zeta_{ij} = \int_0^l \Phi_i(x)\Phi_j(x)dx, \qquad \varrho_{ij} = \int_0^l \Phi_i''(x)\Phi_j(x)dx,\\
&\upsilon_{ij} = \int_0^l \Psi_i'(x)\Phi_j(x)_j(x)dx, \qquad \eta_{ij} = \int_0^i \Psi_i''(x)\Psi_j(x)dx,\\
&\varepsilon_{ij} = \int_0^l \Psi_i(x)\Phi_j'(x)dx, \qquad \varsigma_{ij} = \int_0^l \Psi_i(x)\Psi_j(x)dx.
\end{aligned} \tag{26}$$

To simultaneously solve the above matrix equations, the following transformations are applied:

$$T_1 \cdots T_n \to X_1 \cdots X_n, \qquad Q_1 \cdots Q_n \to X_{n+1} \cdots X_{2n}. \tag{27}$$

By using the above transformations, the obtained ordinary differential equations will be presented as:

$$\boldsymbol{N}\ddot{\boldsymbol{X}} + \boldsymbol{E}\dot{\boldsymbol{X}} + \boldsymbol{\Sigma}\,\boldsymbol{X} = \boldsymbol{\Pi}, \tag{28}$$

in which:

$$\begin{aligned}
&\boldsymbol{N}_{2n\times 2n} = \begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \overline{\boldsymbol{M}} \end{bmatrix}, \qquad \boldsymbol{E}_{2n\times 2n} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0}, \end{bmatrix},\\
&\boldsymbol{\Sigma}_{2n\times 2n} = \begin{bmatrix} \boldsymbol{K} & \boldsymbol{\Delta} \\ \overline{\boldsymbol{K}} & \overline{\boldsymbol{\Delta}} \end{bmatrix}, \qquad \boldsymbol{\Pi}_{2n} = \begin{bmatrix} \boldsymbol{F} \\ \overline{\boldsymbol{F}} \end{bmatrix},
\end{aligned} \tag{29}$$

then again, to represent the above equation in state-space form, the transformation below is needed:

$$\boldsymbol{X}_{2n} = \boldsymbol{D}1_{2n}, \qquad \dot{\boldsymbol{X}}_{2n} = \boldsymbol{D}2_{2n}, \qquad \ddot{\boldsymbol{X}}_{2n} = \dot{\boldsymbol{D}}2_{2n}, \tag{30}$$

while:

$$\boldsymbol{D}_{4n} = \begin{bmatrix} \boldsymbol{D}1_{2n} \\ \boldsymbol{D}2_{2n} \end{bmatrix}. \tag{31}$$

Finally the state space form of the equations is expressed as:

$$\begin{bmatrix} \dot{\boldsymbol{D}}1 \\ \dot{\boldsymbol{D}}2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{N}^{-1}\boldsymbol{\Sigma} & -\boldsymbol{N}^{-1}\boldsymbol{E} \end{bmatrix} \begin{bmatrix} \boldsymbol{D}1 \\ \boldsymbol{D}2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{N}^{-1} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Pi} \\ \boldsymbol{0} \end{bmatrix}. \tag{32}$$

Following the same route, when dealing with the case of the moving sprung mass, the following matrix equation can be resulted:

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} & \underline{\boldsymbol{M}} \\ \boldsymbol{0} & \overline{\boldsymbol{M}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{T}} \\ \ddot{\boldsymbol{Q}} \\ \ddot{\boldsymbol{H}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{C} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \overline{\boldsymbol{X}} & \boldsymbol{0} & \boldsymbol{X} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{T}} \\ \dot{\boldsymbol{Q}} \\ \dot{\boldsymbol{H}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{\Delta} & \boldsymbol{\Omega} \\ \overline{\boldsymbol{K}} & \overline{\boldsymbol{\Delta}} & \boldsymbol{0} \\ \overline{\boldsymbol{\Gamma}} & \boldsymbol{0} & \boldsymbol{\Gamma} \end{bmatrix} \begin{bmatrix} \boldsymbol{T} \\ \boldsymbol{Q} \\ \boldsymbol{H} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F} \\ \overline{\boldsymbol{F}} \\ \underline{\boldsymbol{F}} \end{bmatrix}, \tag{33}$$

in which the matrix components are:

$$\begin{aligned}
&M_{ij} = \rho A \zeta_{ij}, \qquad \underline{M}_{ij} = m\Phi_i(X)\Phi_j(X), \\
&C_{ij} = 2m\dot{X}\Phi_i'(X)\Phi_j(X), \qquad K_{ij} = -\kappa GA\varrho_{ij}, \\
&\Delta_{ij} = \kappa GA v_{ij}, \\
&\Omega_{ij} = m\dot{X}^2\Phi_i''(X)\Phi_j(X) + m\ddot{X}\Phi_i'(X)\Phi_j(X), \\
&F_j = mg\Phi_j(X), \qquad \overline{M}_{ij} = \rho I \varsigma_{ij}, \\
&\overline{\Delta}_{ij} = \kappa GA\varsigma_{ij} - EI\eta_{ij}, \\
&\overline{K}_{ij} = -\kappa GA\varepsilon_{ij}, \qquad \overline{F}_i = 0, \\
&A_{ij} = m\Phi_i(X)\Phi_j(X)\delta_{ij}, \qquad \overline{X}_{ij} = -c\Phi_i(X)\Phi_j(X)\delta_{ij}, \\
&\Gamma_{ij} = k\Phi_i(X)\Phi_j(X)\delta_{ij}, \qquad X_{ij} = c\Phi_i(X)\Phi_j(X)\delta_{ij}, \\
&\overline{\Gamma}_{ij} = -k\Phi_i(X)\Phi_j(X)\delta_{ij}, \qquad \underline{F}_i = 0.
\end{aligned} \tag{34}$$

To represent Eq. (33) in state-space form, the following transformations are performed:

$$\begin{aligned} &T_1 \cdots T_n \to X_1 \cdots X_n, \qquad Q_1 \cdots Q_n \to X_{n+1} \cdots X_{2n}, \\ &H_1 \cdots H_n \to X_{2n+1} \cdots X_{3n}, \end{aligned} \tag{35}$$

$$\boldsymbol{X}_{3n} = \boldsymbol{D}1_{3n}, \qquad \dot{\boldsymbol{X}}_{3n} = \boldsymbol{D}2_{3n}, \qquad \ddot{\boldsymbol{X}}_{3n} = \dot{\boldsymbol{D}}2_{3n}. \tag{36}$$

The resulted state-space form of the equations in both cases is written as:

$$\dot{\boldsymbol{D}}(t) = \overline{\boldsymbol{A}}(t)\boldsymbol{D}(t) + \overline{\boldsymbol{E}}(t)\boldsymbol{\Pi}(t), \tag{37}$$

where:

$$\dot{\boldsymbol{D}}(t) = \begin{bmatrix} \dot{\boldsymbol{D}}1 \\ \dot{\boldsymbol{D}}2 \end{bmatrix}, \qquad \overline{\boldsymbol{A}}(t) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{N}^{-1}\boldsymbol{\Sigma} & -\boldsymbol{N}^{-1}\boldsymbol{E} \end{bmatrix}, \qquad \overline{\boldsymbol{E}}(t) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{N}^{-1} & \boldsymbol{0} \end{bmatrix}. \tag{38}$$

The numerical procedure below can be applied to solve the set of the first order differential equation with time-varying coefficients [25]:

$$\boldsymbol{D}(t_{k+1}) = \overline{\boldsymbol{A}}_1(t_k)\boldsymbol{D}(t_k) + \overline{\boldsymbol{E}}_1(t_k)\boldsymbol{\Pi}(t_k), \tag{39}$$

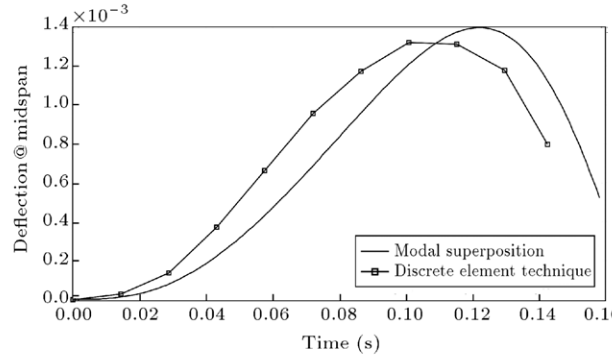

Figure 3: Discrete element technique versus the approach of this paper in estimating time histories of displacements of beam subjected to moving mass.

where:

$$\overline{\boldsymbol{A}}_1(t_k) \cong \boldsymbol{e}^{\overline{\boldsymbol{A}}(t_k)\Delta(t_k)}, \tag{40}$$

$$\overline{\boldsymbol{E}}(t_k) \cong [\overline{\boldsymbol{A}}_1(t_k) - \boldsymbol{I}]\overline{\boldsymbol{A}}^{-1}(t_k)\overline{\boldsymbol{E}}(t_k). \tag{41}$$

3. Parametric study

Before proceeding with the parametric study, the time histories estimated through the approach adopted in this paper and discrete element technique are compared to ensure the correctness of the equations. In this regard, Figure 3 illustrates a comparison of the time histories of a numerical example in the Ref. [15]. It is observed that, the rhythm of both solutions is fairly similar. Furthermore, there are about 15 up to 25% differences between the two different solutions. These differences between the two plots are due to the fact that the "Discrete Element Technique", is an approximate solution of the problem; on the contrary, the approach adopted in the current research is an analytical–numerical advancement.

To accomplish the task of parametric study, a simply supported beam is studied in order to assess the effect of variation of each parameter on the dynamic response of the structural system. To accomplish this, four non-dimensional parameters namely α, β, γ and ξ are considered. The first parameter, α, denotes the speed ratio; β represents the mass ratio; γ stands for frequency ratio and ξ is the damping ratio, as described by the following formula:

$$\alpha = \frac{V}{V'}, \qquad \beta = \frac{m}{\rho AL}, \qquad \gamma = \frac{\omega_v}{\omega_b}, \qquad \xi = \frac{c}{2m\omega_v}, \tag{42}$$

in which:

$$\omega_b = \pi^2\sqrt{EI/\rho A l^4}\left(1 - \frac{1}{2}\left(\frac{r\pi}{l}\right)^2\left(1 + \frac{E}{kG}\right)\right), \qquad \omega_v = \sqrt{\frac{K}{m}}, \qquad V' = \frac{L\omega_b}{\pi}. \tag{43}$$

The parametric study is carried out via a numerical example in order to provide a comparison among three load cases of moving mass, moving sprung mass and moving load. According to [19], the nth natural mode shape functions of deflection and rotation of a simply supported Timoshenko beam are $A_n \sin(n\pi x/l)$ and $B_n \cos(n\pi x/l)$, respectively, where, A_n and B_n denote the contribution of the nth deflection and the rotational mode, respectively. The results are derived from the analysis of a beam with the span length of 50 m, cross sectional area of 2 m^2, elastic modulus of 3.34×10^{10} N/m^2, shear modulus

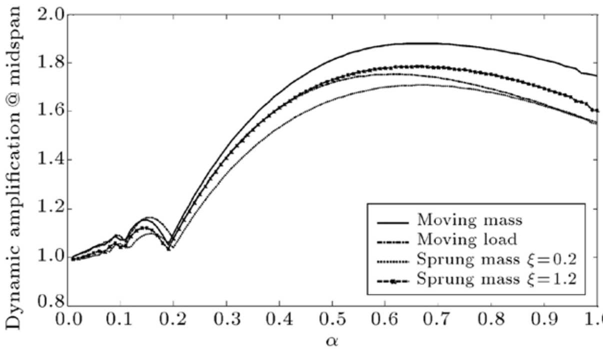

Figure 4: The maximum midspan deflection under the effect of moving mass; $\gamma = 0.5$ and $\beta = 0.1$.

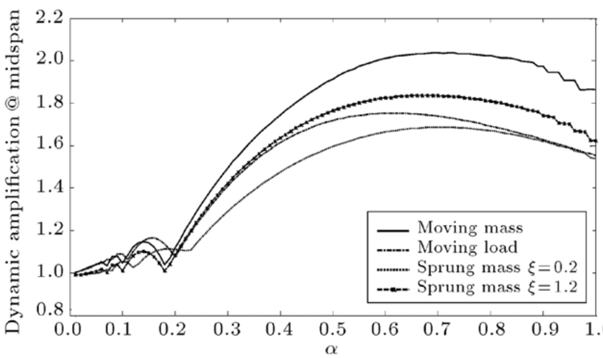

Figure 5: The maximum midspan deflection under the effect of moving mass; $\gamma = 0.5$ and $\beta = 0.2$.

of 1.34×10^{10} N/m^2, mass per unit volume of 2400 kg/m^3 and inertial moment of 1.042 m^4. However, the results are valid for any other values which yield corresponding non-dimensional parameters. In all the figures, the horizontal axis stands for the speed ratio, α, instead the ordinate represents the corresponding non-dimensional maximum dynamic deflection of midspan of the beam; so that it can be called dynamic amplification due to the moving loads. The results are plotted for three values of mass and frequency ratio, also two damping ratios: one over critical and one under critical damping considered to permit the spring-dashpot system to vibrate in two different regimes.

Figures 4–6 illustrate the results of the numerical analysis for the mass ratios of 0.1, 0.2 and 0.3, respectively. In the aforementioned figures, the frequency ratio is set as 0.5. Furthermore, it is observed that as the mass ratio increases, the discrepancy among the responses of the system becomes more significant for the cases when the inertial effects of the mass are included and in the case they are excluded, respectively. Concerning the case in which a spring-dashpot system is connecting the mass to the beam, it is observed that apart from a limited interval of the speed ratio, the magnitude of the response of the beam to a moving sprung mass with under critical damping ratio is lower than the response to moving load.

Figures 7–12 present the results of the analysis for the same assumption made for Figures 4–6, and only the value of the frequency ratio is varied to see if there is a pattern for values equal to 1, below 1 and above it. It is observed that there is no significant sensitivity to the frequency ratio; however, as the frequency of the vehicle increases with respect to the beam, the response of the beam to moving sprung mass approaches the

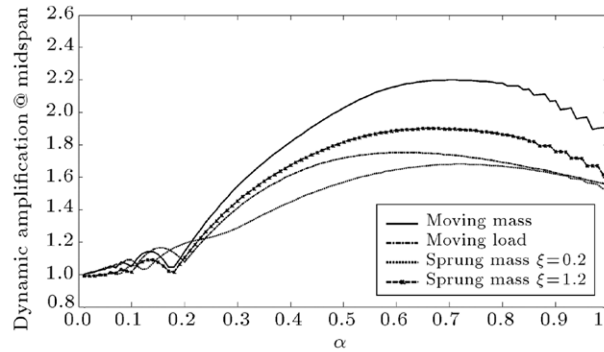

Figure 6: The maximum midspan deflection under the effect of moving mass; $\gamma = 0.5$ and $\beta = 0.3$.

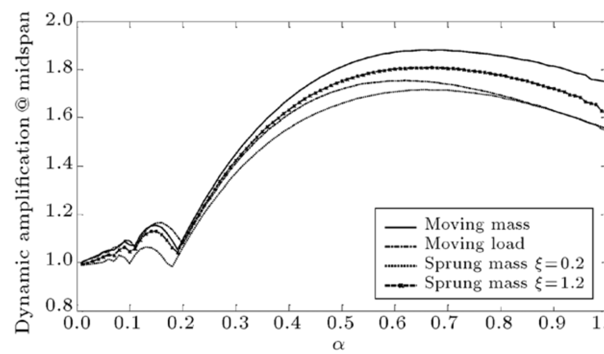

Figure 7: The maximum midspan deflection under the effect of moving mass; $\gamma = 1$ and $\beta = 0.1$.

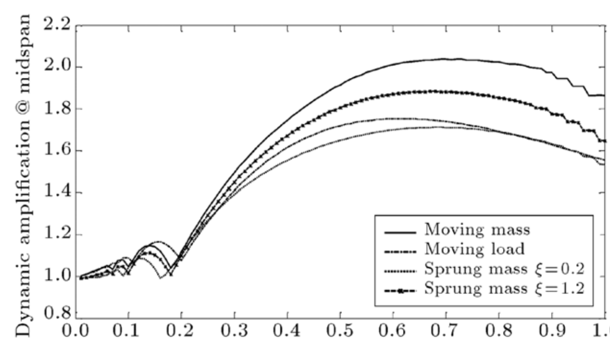

Figure 8: The maximum midspan deflection under the effect of moving mass; $\gamma = 1$ and $\beta = 0.2$.

moving mass case. It is further observed that the key parameters are mass and damping ratios.

To analyze the effects of aforementioned effective parameters, two other plots are prepared and presented. The maximum deflection of midspan for various amounts of ξ in contrast to the plots concerning moving mass and moving load case are plotted in Figure 13. Instead, the results of the analysis for a variation in the mass ratio are presented in Figure 14.

In all the plotted figures, it is observed that the results of the numerical analysis for the systems with damping ratio above critical damping are between moving load and moving mass results, where moving mass results are an upper bound for the problem. While for the oscillations below critical damping ratio, the results appear to be less than moving load problem.

For damping ratios more than the critical damping, the response of the system is greater than the moving load. At

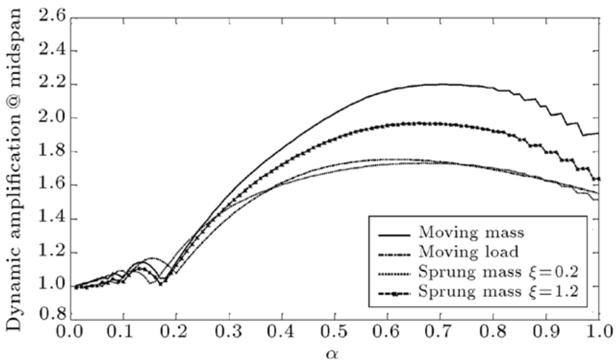

Figure 9: The maximum midspan deflection under the effect of moving mass; $\gamma = 1$ and $\beta = 0.3$.

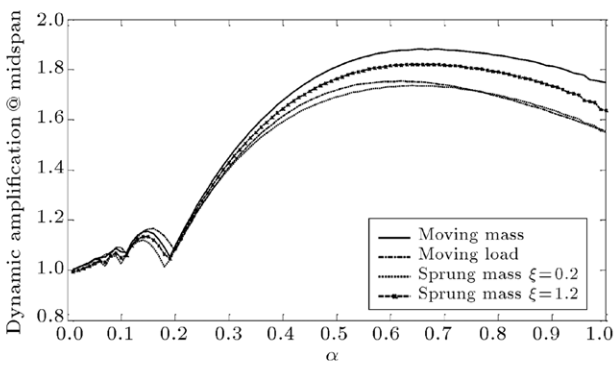

Figure 10: The maximum midspan deflection under the effect of moving mass; $\gamma = 1.5$ and $\beta = 0.1$.

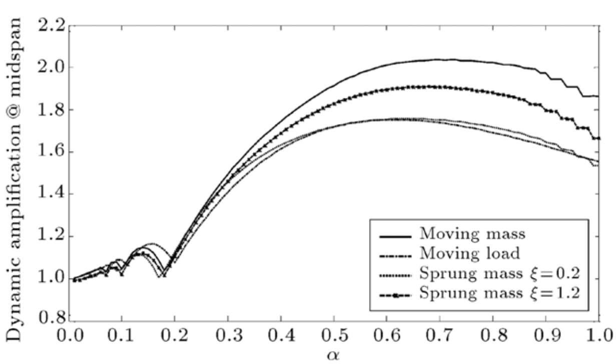

Figure 11: The maximum midspan deflection under the effect of moving mass; $\gamma = 1.5$ and $\beta = 0.2$.

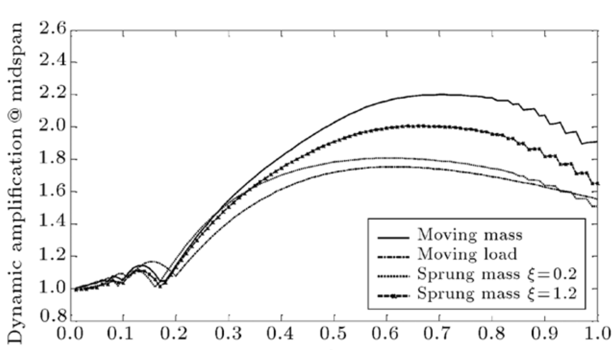

Figure 12: The maximum midspan deflection under the effect of moving mass; $\gamma = 1.5$ and $\beta = 0.3$.

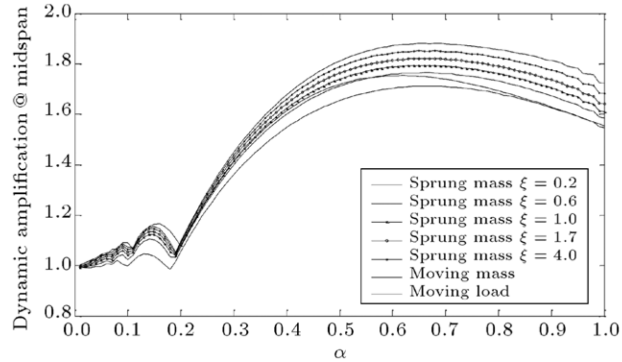

Figure 13: The maximum midspan deflection under the effect of moving mass, moving sprung mass and moving load.

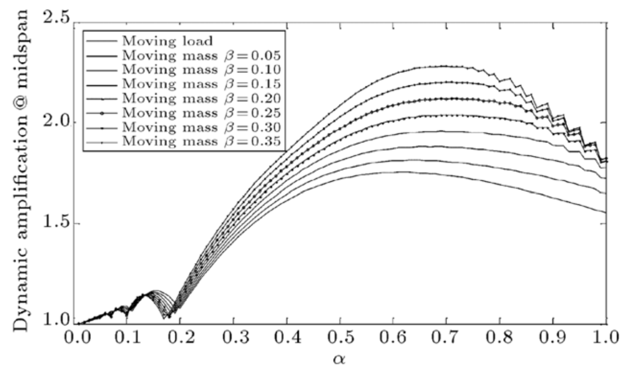

Figure 14: The maximum midspan deflection under the effect of moving mass and moving load.

the same time, it is further understood that for this range of damping ratio, even for large ratios of moving mass, and speed parameter, the moving load is primarily an upper bound for the problem.

Remarks on the theory

- Contribution of each vibration mode is taken into account, and the effect of each individual mode can be investigated.
- Series expansion of dynamic response converges very quickly. It was observed that two modes of vibration are sufficient to capture the dynamics of the system.
- The effect of accelerating motion of the travelling mass can be investigated.
- The effect of suspension system properties of one degree of freedom vehicle model can be investigated.
- The new formulation of the equations permits further studies of the problem when the velocity of the moving load is not constant.

4. Conclusion

The dynamic response of a moving sprung mass is primarily compared to the dynamic responses of moving mass and moving load; it is understood that, moving mass assumption results in the greatest values for amplification factor compared to sprung mass cases and therefore is an upper bound solution to the problem. As for the contribution of this study, authors claim that for sub-critical values of suspension system properties, the moving load results are an upper bound for moving sprung mass results. It means that while dealing with a real problem case, an analysis made by neglecting suspension system and also inertia of the moving mass will be still a conservative estimate of the responses of the system.

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